englinks

Grade 12 Physics Review Workbook

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Course Fundamentals

Prefixes

A single unit of measurement cannot often describe the entire physical universe, so prefixes are added to the measurements to either increase or decrease their size. For example, when measuring length, a quark is found to be only 10^{-18} m, an attometer, while a light year on the other hand is as large as 10^{15} m, a petameter. A table of measurement prefixes is provided below.

Prefix	Symbol	Order of Magnitude
Deka-	da	10^1
Hecto-	h	10^2
Kilo-	k	10^3
Mega-	M	10^6
Giga-	G	10^9
Tera-	T	10^{12}
Peta-	P	10^{15}

Prefix	Symbol	Order of Magnitude
Deci-	d	10^{-1}
Centi-	c	10^{-2}
Mili-	m	10^{-3}
Micro-	µ	10^{-6}
Nano-	n	10^{-9}
Pico-	p	10^{-12}
Femto-	f	10^{-15}

Scalar vs. Vector Quantities

A scalar quantity has only a value, whereas a vector has both a value and a direction. While you should be familiar with scalar operations of addition, subtraction, multiplication and division, vectors have a slightly different system of operations. These include addition, subtraction, the dot product, and the cross product.

Scalar	**Vector**
Mass	Weight
Distance	Displacement
Speed	Velocity
Energy	Force
Time	Acceleration

Since vectors have direction, we often write vectors in terms of their components, where each component is the magnitude of the vector in a certain direction. By looking at vectors with 2 components or less, we can easily represent them graphically. Larger vectors are still easier to represent in equations and are denoted in these ways.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad \left\langle \begin{matrix} a \\ b \\ c \end{matrix} \right\rangle \qquad a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

When given in one of these forms, the length or magnitude of the vector can be calculated by taking the square root of the sum of all terms in the vector squared. The magnitude of a vector is denoted with two vertical lines on each side.

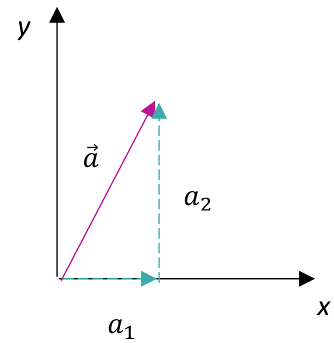
$$\left\|\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}\right\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} \qquad [1]$$

This can be easily demonstrated using a 2-D vector, and you will see that the method is the same as Pythagoras' theorem.

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\|\vec{a}\|^2 = a_1^2 + a_2^2$$

$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2}$$



Vector Addition

To add vectors, you add the like terms in the two vectors as seen in Equation 2. Like scalar addition, vector addition is both commutative and associative. So, the order in which you add vectors does not matter:

$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \end{pmatrix} \qquad [2]$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_1+a_1 \\ b_2+a_2 \end{pmatrix} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \end{pmatrix} \qquad [3]$$

And, when adding multiple vectors and grouping, the order does not matter either:

$$\left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}\right] + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a_1+b_1+c_1 \\ a_2+b_2+c_2 \end{pmatrix} \qquad [4]$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \left[\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}\right] = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 + c_1 \\ a_2 + b_2 + c_2 \end{pmatrix}$$

Vector Subtraction

Vector subtraction is the same process as adding vectors, where terms in the same position are combined. However, we can think of subtracting a vector as adding a negative vector. A vector is a value with a direction, so a negative vector has the same value in the opposite direction. To write this numerically, simply change the signs of all terms in the vector. By taking the opposite of each component of the vector, the direction of the entire vector is reversed, and it is considered a negative vector. For example:

$$\vec{a} = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} \qquad -\vec{a} = \begin{pmatrix} -4 \\ 5 \\ -3 \end{pmatrix}$$

By writing negative vectors this way, vector subtraction is just as easy as addition:

$$\vec{a} - \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} -b_1 \\ -b_2 \end{pmatrix} = \begin{pmatrix} a_1-b_1 \\ a_2-b_2 \end{pmatrix} \qquad [5]$$

Dot Product

The dot product is an operation that uses two vectors as an input and the output is a real number. Geometrically, Equation 6, defines the dot product of any two vectors where θ is the angle between the vectors. To compute the dot product, you take the sum of the products of corresponding terms between the vectors.

$$\vec{a} \cdot \vec{b} = \|a\|\|b\| \cos\theta \quad [6]$$

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \quad [7]$$

By taking the dot product of a vector and itself, as seen in Equation 8 below, the result looks very similar to Equation 1, but squared.

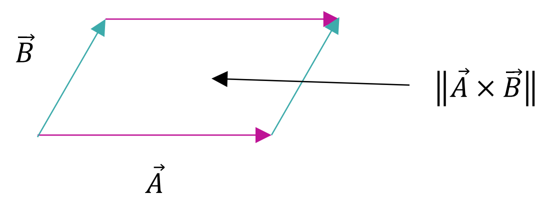
$$\vec{a} \cdot \vec{a} = a_1^2 + a_2^2 + \cdots + a_n^2$$

$$\|\vec{a}\|^2 = \vec{a} \cdot \vec{a} \quad [8]$$

Cross Product

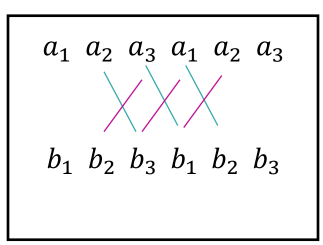
The cross product is our last vector operation. Like addition and subtraction, its output is also a vector. Taking the cross product of two vectors tells us two useful things. First, the magnitude of the cross product is the area of the trapezoid made by the two vectors. Second, when using Equation 9, the unit vector at the end, $\hat{a}$, will tell you all vectors that are perpendicular to the plane of both vectors.

$$\vec{A} \times \vec{B} = \|\vec{A}\|\|\vec{B}\| \sin\theta \, \hat{a} \quad [9]$$



One way to quickly solve for the cross product involves writing out both vectors sideways to visualize the operation better. Take vectors $\vec{A}$ and $\vec{B}$. $\vec{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\vec{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$. By listing the terms twice and drawing these crosses like the image on the right, it is easier to memorize the formula on the left.

$$\vec{A} \times \vec{B} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$



Vector Practice Problems

Two teams are playing tug of war. The team on the left pulls on the horizontal with a total force of 290 N. The team on the right pulls with a greater force of 310 N at an angle of 24^{o} above the horizontal.

a. Which team is more likely to win the tug of war?

b. What is the net force on the rope?

A plane leaves YYZ headed 200 km NE, after stopping here it travels and 75 km east and takes a final trip 130 km 30^{o} south of west.

a. Using a Cardinal system, find the North/South and East/West components of the flights and sketch them

b. How far is the plane from its starting point?

c. How far has the plane travelled in total?

Find the angle between $\vec{a}$ and $\vec{b}$ given $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$. Then, imagine a third line closed the ends of the vectors to make a triangle, what is the area?

Kinematics

Kinematics is the branch of physics that deals with the motion of objects without looking at the forces that causes such motion. The basics of motion involve four key variables: position, *x*, velocity, *v*, acceleration, *a*, and time, *t*. The definitions of average velocity and acceleration are below:

$$\bar{v} = \frac{\Delta x}{\Delta t} \qquad [10]$$

$$\bar{a} = \frac{\Delta v}{\Delta t} \qquad [11]$$

Kinematics focuses on objects that are free falling, meaning the only external force acting on the object is the <u>constant</u> acceleration of gravity. The equations below can be used whenever an object is in free fall. By identifying which variables are given, and which values you are solving for, it is easy to select the right equation to use.

$$v = at + v_0 \qquad [12]$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad [13]$$

$$x = x_0 + vt - \frac{1}{2}at^2 \qquad [14]$$

$$x = x_0 + \frac{1}{2}(v + v_0)t \qquad [15]$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [16]$$

Kinematics Practice Problems

Two planes are flying at the same altitude. Plane A is flying at 400 km/hr due North while plane B flies at 700 km/hr 20^{0} South of West. At the same instant, Plane A drops a load of cargo totalling 2.5 tons and plane B drops cargo weighing 2 tons.

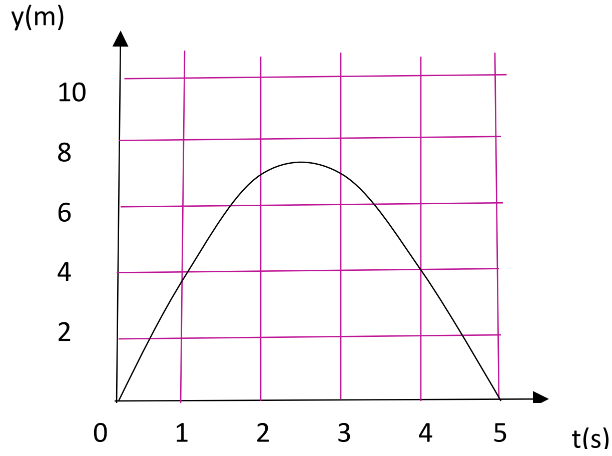
a. Which planes cargo will hit the ground first?

b. If both planes continue to fly at the same velocity and on the same course after dropping their cargo, where will the loads lands relative to their respective planes?

John and Kerry are standing on Planet A and B respectively holding the same gun. Planet A has an acceleration due to gravity twice as strong as planet B's. What is the ratio $\frac{Planet\ A}{Planet\ B}$ of the distance travelled by the bullets on the two planets?

A) $\frac{1}{2}$
B) $\frac{1}{\sqrt{2}}$
C) 1
D) $\sqrt{2}$
E) 2

You throw a ball straight into the air and catch it in your hands again. The graph on the rights shows the vertical displacement of the ball over time. Use the graph to answer the questions below:



When is the ball at its peak height?

a) 0.5s
b) 1.3
c) 2.5s
d) 4.2s
e) 5.0s

What is the total distance travelled by the ball?

a) 5m
b) 7m
c) 10m
d) 14m

What is the initial velocity of the ball?

a) 0m/s
b) 2m/s
c) 4m/s
d) 8m/s

A projectile is fired from a cliff 50m above ground level. The projectile is fired at 35^{o} above the horizontal with an initial velocity of 10m/s at a target is placed on the ground 30m away from the base of the cliff.

a. What is the maximum height the projectile will reach before beginning its descent?

b. Will the projectile hit the target on the ground?

c. If the target is not hit, what can be changed about the firing settings that would guarantee a hit?

[EXTRA PRACTICE] **A player shoots a basketball into a hoop. The player shoots at 60^{o} above the horizontal from 1.8m above the ground into the basket 3m above ground. The ball reaches the peak of its arc at 0.619s at a maximum height of 3.675m.**

a. What was the initial velocity of the shot leaving the players hands?

b. How far away did the player shoot from?

Dynamics

Dynamics is a branch of physics that deals with the motion of objects in relation to the factors that affect them: force, mass, momentum, and energy. When analyzing forces on an object we look at Newton's laws:

I. An object in motion stays in motion unless acted upon by an outside force
II. F=ma
III. Every action has an equal and opposite reaction

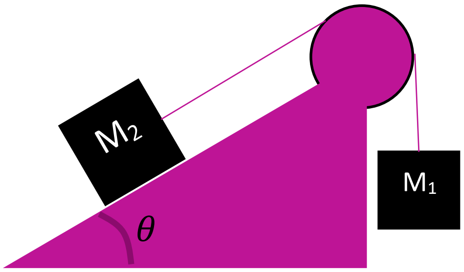
The first law explains that an object's velocity cannot be influenced by anything other than an outside force. Therefore, if an object is standing still it will not move until pushed and if an object were floating through space, it would never stop until an obstacle got in its way.

The second law relates force to acceleration through mass. It states that, the force it takes to accelerate an object is directly proportional to that objects mass.

The third law focuses on reaction pairs. If object A applies a force on object B, object B is applying that same force on object A in the opposite direction. This shows that no collisions or interactions are one sided. Even when jumping off the ground, the Earth experiences as much force from your legs as your legs experience from the Earth.

Dynamics Practice Problems

Two blocks are connected over an ideal pulley by a massless string. One block is hanging vertically with a mass of 12 kg, the other lies on a frictionless inclined plane and has a mass of 18 kg.



a. Draw FBDs of the forces acting on both boxes.

b. What angle should the ramp be at for the system to be in equilibrium?

c. If the ramp were positioned at an angle $\theta = 30°$ above the horizontal, what is the total acceleration?

You are standing in a moving elevator while weighing your lunch on a scale. Your sandwich has a total mass of 250g, and the scale reads 2.92 N.

a. Draw an FBD of the forces acting on your lunch.

b. What is the actual weight of your lunch?

c. Determine the magnitude and direction of the net force on your lunch.

d. Determine the magnitude and direction of the acceleration of the elevator

e. In this scenario, the elevator is moving upwards. Is the elevator coming to a stop, at a constant speed, or still reaching its top velocity?

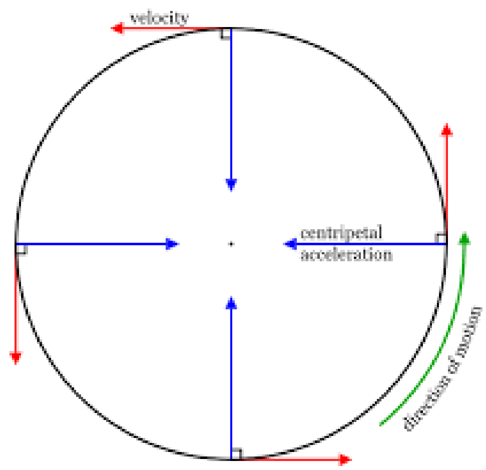
Circular Motion

When moving in a circular arc, we can describe motion in terms of rotation rather than displacement. The linear variables of position, velocity and acceleration are now replaced with angle (in radians), angular velocity, and angular acceleration. Since we are looking at objects moving in perfect circles, all linear and angular variable are related through the radius of the circle.

$$x = \theta r \qquad v = \omega r \qquad a = \alpha r$$

In general, objects tend to accelerate in the direction of motion to increase or decrease speed. However, in circular motion, all acceleration is directly perpendicular to the direction of motion. This is called centripetal acceleration. By applying a constant force perpendicular to the direction of travel, an object will turn without changing speed. This acceleration can be calculated using either the linear or angular velocity of the spinning object along with the radius of the path.

$$a_c = \frac{v^2}{r} = \omega^2 r \qquad [17]$$



A cowboy spins his lasso above his head in preparation of catching a bull. The lasso spins in a full circle, of diameter 2 meters, 3 times per second.

a. What is the period of the motion of the rope?

b. What is the acceleration of the loop on the end of the lasso?

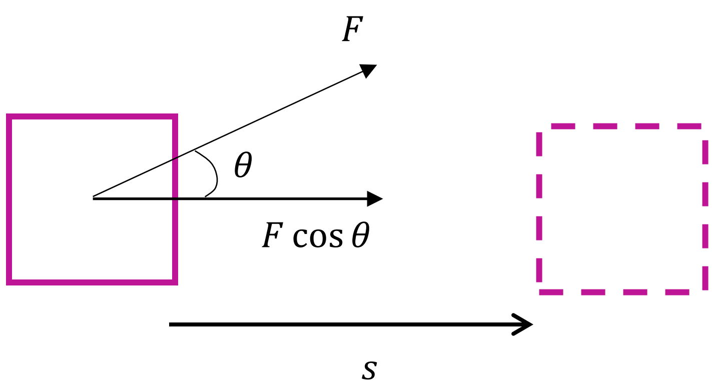
c. Assuming the end of the lasso has a mass of 1.5 kg, and the rope's mass is negligible, what is the magnitude of the force experienced by the rope?

Work and Energy

Energy can never be created nor destroyed, it only transfers forms. Conservation of energy is the idea that in any process, the energy into a system is equal to the energy out of the system. We will mainly deal with object transferring energy between kinetic and potential, or a loss of energy to the surroundings. The total energy is the sum of the kinetic, potential, and all other types of energy in a system.

Work is the process of transferring energy into the motion of an object through the application of a force on the object. Although work is scalar, so it has no direction, a force is said to do positive work on an object if it is applied in the same direction as the motion of the object and negative if acting in the opposite direction. Forces applied perpendicular to motion do not do work on the object. Equation 18 shows how to determine the amount of work done as a function of force, displacement, and angle of the applied force.

$$W = Fs \cdot cos\theta \qquad [18]$$



Finding the net work on an object tells us about the kinetic energy of a system. The work-energy theorem, as seen in Equation 19, states that net work is equal to a change in kinetic energy and is true for motion with non-uniform acceleration. This makes solving problems with multiple stages of motion much easier.

$$W_{net} = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad [19]$$

Positive work adds kinetic energy to a system, while negative work takes it away. An object will gain or lose KE due to other bodies applying forces to it. IN some collisions, that KE is lost forever into other forms of energy, this is a nonconservative force, such as friction, or drag. Other forces allow all the KE to be transferred back into the original object. These are conservative forces such as gravity, spring force, and in most cases magnetism.

Potential energy is energy related to the position of an object. For example, objects that are further from the center of a gravitational field have greater PE than objects that are closer. This is because they have a greater potential to gain kinetic energy as they accelerate towards the Earth's ground. Potential energy is always measured from a reference point, in this case Earth's ground. The reference point chosen does not matter, as long as it stays the same throughout the problem, since only a change in potential energy, ΔPE, is observed. Generally, work done by conservative forces is equal but opposite to the change in potential energy of the system.

$$-W_c = \Delta PE \qquad [20]$$

Using the concepts of conservative and nonconservative work the work-energy theorem can be rewritten as Equation 21. By also substituting Equation 20 for W_c, we arrive at the more general Equation 22 which can be used to solve most problems regarding work and energy.

$$W_c + W_{nc} = \Delta KE \qquad [21]$$

$$W_{nc} = \Delta KE + \Delta PE = \Delta E \qquad [22]$$

Work and Energy Practice Problems

You are driving down the road at 50 km/h and you see an animal 30 m away in the road. Will you do more work by staying straight and braking in time or turning (in a constant circular motion) to avoid the animal?

Thomas is pulling a sled behind him across frictionless ice. He pulls upwards and rightwards at an angle of 37^{o} above the horizontal. If Thomas pulls with 65 N of force for 1.5km, how much work is done on the sled in Joules?

A 15 kg block slides 5 m down an inclined plane at 30^{0} onto a spring with spring constant 70 N/m. The coefficient of friction of the block on the ramp is 0.5. Upon reaching the spring it compresses and launches the box back up the ramp.

a. How far did the spring compress?

b. How far up the ramp did the block slide from the end of the spring?

Momentum

Momentum is mass in motion, and all moving objects have momentum. Object's with greater speed and more mass have more momentum compared to others.

$$p = mv \qquad [23]$$

A change in momentum is called impulse and is measured as a force times a time interval. Impulse is not the same as momentum itself, but a word for the increase or decrease, or change in direction of momentum.

$$Impulse = p_f - p_i = \Delta p \qquad [24]$$

$$\Delta p = m\Delta v = F\Delta t \qquad [25]$$

The law of conservation of momentum states that momentum in a closed system is constant. Therefore, the product of mass and velocity in the system is constant and always equal. It is important to identify what is included in the system at the start of a problem and remember that, in a closed system, nothing enters, and nothing exits.

$$p_i = p_f = m_i v_i = m_f v_f \qquad [26]$$

Objects collide in either an elastic or an inelastic collision. An elastic collision is one in which there is no loss of kinetic energy. In an inelastic collision, some of the kinetic energy is transferred to other forms of energy such as heat or sound. Conservation of momentum, Equation 26, stays true throughout all collisions.

Momentum Practice Problems

You are riding your bike at a constant speed with a friend on the back who is roughly the same weight as you. As you are going down the street your friend falls off the back so that they are no longer in contact with the bike. How does your speed change at the instant your friend falls off the bike?

A. Speeds up
B. Slows down
C. Same speed
D. Not enough information to tell

A bullet with a mass of 50g, is shot vertically into a 10 kg block of wood. As the bullet strikes the block it has a speed of 600 m/s. Assume the bullet sticks into the block in an inelastic collision after contact.

a. What is the speed of the system after contact?

b. What was the block's maximum displacement after the contact?

Tim and Sarah are playing squash at the ARC. Sarah hits the 25 g ball directly at the wall with a speed of 60 m/s. After striking the wall it returns in the same direction with a speed of 45 m/s. What was the total impulse due to the collision of the ball with the wall?

On receiving the ball, Tim's racket makes contact for 0.04 seconds and returns the ball directly towards the wall at a speed of 56 m/s. Was the total impulse of this collision greater or lesser than that of Sarah's ball hitting the wall? How much force did Tim exert on the ball while striking it?

References

Some material from this workbook has been adapted from the QEng Prep: Physics course created by the Faculty of Engineering and Applied Science at Queen's University.

H. D. Young and R. A. Freedman, Sears and Zemansky's University Physics, 14th Edition. University of California, Santa Barbara: Pearson, 2016.